Dear Calculus BC student,

I hope that you're all enjoying your first few days of summer! Here's something that will make it a little more fun! Enclosed you will find a packet of review questions that you should complete before the first day of classes. This packet covers mainly Algebra I, Algebra II, and Precalculus skills. These are skills that we will work on having down pat so that you won't even have to think about, "how do I solve this problem?" The packet will be graded upon your return to school and will be worth 4 homework assignments (a week's worth of assignments). Solutions are also attached, but I expect to see full work for each problem.

There will be a quiz covering this material on the second or third day of classes.

Many of you may want to complete this as soon as possible and have it over and done with. If this is the case, please be sure to review your solutions in the days before school starts. The aim of this summer work is to keep your mathematical mind from rusting in the months that you'll spend away from school. We'll be hitting the ground running in August and you don't want to be out of breath. Enjoy!

E-mail any questions: jbowman@keyschool.org

See you all in August!

Mrs. Bowman

If you come across a topic with which you are uncomfortable, you may find the following websites useful:

http://www.khanacademy.org http://www.coolmath.com/ http://www.purplemath.com/

Help is also available in your Precalculus notebook or textbook. Did you save your notes?

The following problems involve the skills that you will be using throughout the year in Calculus BC. Let's see what you remember! We will answer some questions on the first day of class, but your goal is to answer your own questions before class starts. You can phone a friend, google, use khanacademy.org, use your Precalculus notes/textbook etc. Be honest with yourself and how comfortable you are with these skills. We'll discuss in August!

Part 1: Simplifying

Simplify each expression completely.

1. $\dfrac{\dfrac{1}{x+4}-\dfrac{1}{4}}{x}$

2. Rationalize the numerator: $\dfrac{\sqrt{x-1}+\sqrt{x}}{x}$

3. $\dfrac{x^2-5x-6}{x+1}$

4. $\dfrac{(x-3)^2-(-1)^2}{x-2}$

5. $\dfrac{\sin^2 x+\cos^2 x}{\cos x}$

6. $\dfrac{\sin(2\theta)-\cos\theta}{1-2\sin\theta}$

7. Expand: $\log\left(\dfrac{x^2(x-1)^3}{2-x}\right)$

8. Condense: $\dfrac{1}{2}\left[3\ln(x+1)+\ln 5-2\ln x\right]$

9. $(2x)^4$

10. $\dfrac{t}{t^{1/3}}$

11. $\dfrac{3}{x-1}+\dfrac{x}{x+2}$

12. $\dfrac{\left(1+x^2\right)^{1/2}-x^2\left(1+x^2\right)^{-1/2}}{1+x^2}$

13. $\dfrac{8!}{6!}$

14. $\dfrac{10!}{2!8!}$

15. $\dfrac{(n+1)!}{n!}$

16. $\dfrac{(2n+1)!}{(2n+3)!}$

17. Factor the following expressions completely.

a.) $4x^2-81$

b.) $8x^3+125$

c.) $4x^3-8x^2-25x+50$

d.) $2x^{3/4}-20x^{-1/4}$

18. Write the sum using sigma notation.

a.) $1+x+x^2+x^3+\cdots+x^{17}$

b.) $4+4x^2+4x^4+4x^6+\cdots+4x^{40}$

19. Rewrite the trigonometric expression: $\tan\left(\arccos\left(\frac{3}{x}\right)\right)$ as an algebraic expression.

Part II: Evaluating

1. Fill in **and memorize** the values in the following table. You should be able to come up with these values without having to draw the unit circle each time.

Angle (degree)	**Angle (radians)**	**sin(x)**	**cos(x)**	**tan(x)**
0				
30				
45				
60				
90				

2. Find **exact** values for each of the following trigonometric expressions.

a.) $\cos\left(\frac{\pi}{2}\right)$ b.) $\sin\left(\frac{11\pi}{6}\right)$ c.) $\sin\left(-\frac{13\pi}{4}\right)$

d.) $\cos^{-1}\left(\frac{1}{2}\right)$ e.) $\sin^{-1}\left(-\frac{\sqrt{2}}{2}\right)$ f.) $\tan^{-1}\left(\sqrt{3}\right)$

g.) $\sin\left(\arctan\frac{3}{4}\right)$ h.) $\cot\left(\arcsin\frac{5}{13}\right)$

3. Perform the following polynomial division:

a.) $\left(x^5-4x^4+x^3-7x+1\right)\div\left(x+2\right)$ using synthetic division

b.) $\left(x^5-x^4+x^3+2x^2-x+4\right)\div\left(x^3+1\right)$ using long division.

4. Calculate the volume of each:

a.) Outer radius: 5 cm, Inner radius: 2 cm, height: 10 cm

b.) Cone with height: 8 in, Base Circumference of 10π in

5. Evaluate the following sums:

a.) $\sum_{n=0}^{5}\left(n^2+2n-1\right)$

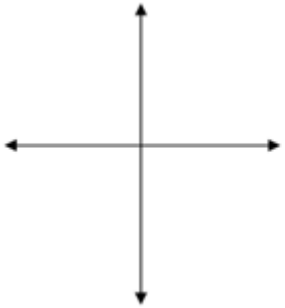
b.) If $\sum_{n=0}^{\infty}\left(\frac{1}{2}\right)^n=2$, find $\sum_{n=2}^{\infty}\left(\frac{1}{2}\right)^n$.

c.) If $\sum_{n=5}^{\infty}4\left(\frac{2}{3}\right)^n=\frac{128}{81}$, find $\sum_{n=2}^{\infty}4\left(\frac{2}{3}\right)^n$.

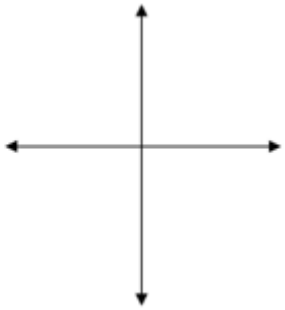
Part III: Graphing

1. Sketch each of the following functions and state their domain and range. Sketch any asymptotes. You should have these functions and their characteristics memorized.

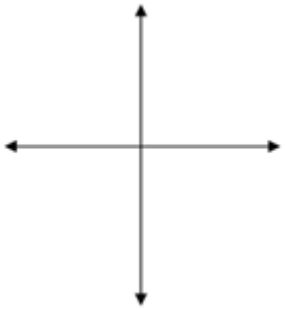
$y = x$



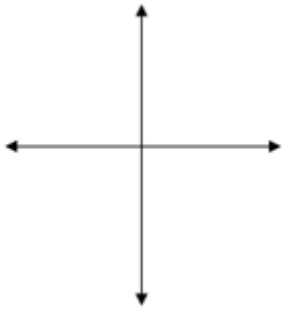
$y = |x|$



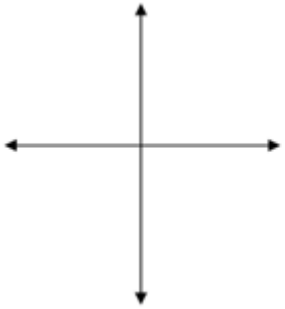
$y = x^2$



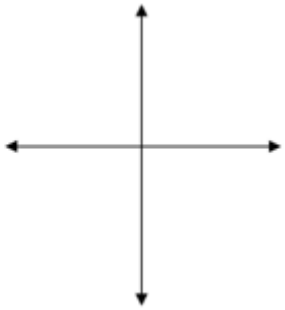
$y = \sqrt{x}$



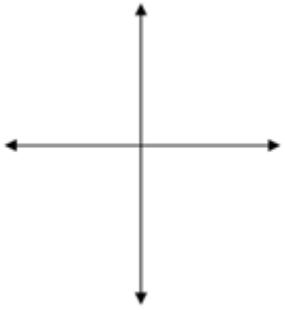
$y = x^3$



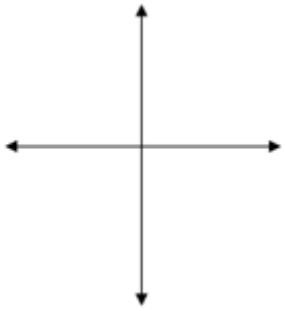
$y = \sqrt[3]{x}$



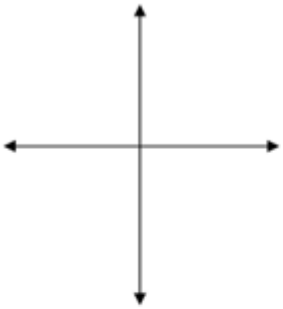
$y = \frac{1}{x}$



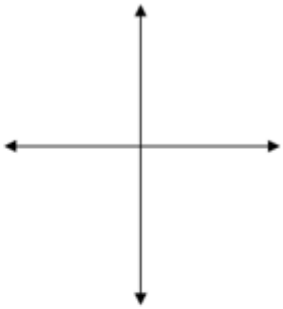
$y = \frac{1}{x^2}$



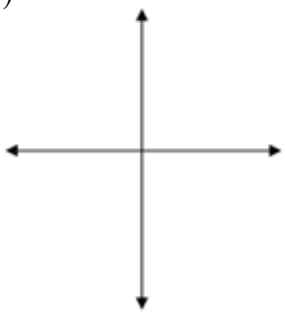
$y = e^x$



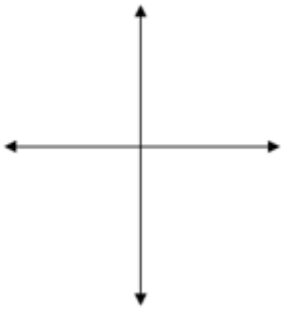
$y = \ln(x)$



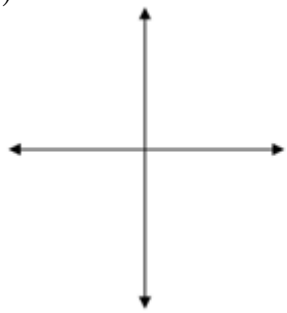
$y = \sin(x)$



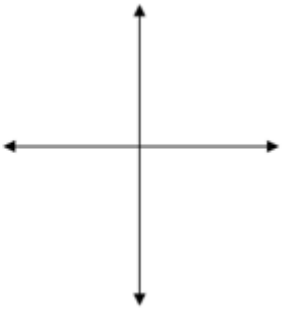
$y = \cos(x)$



$y = \tan(x)$

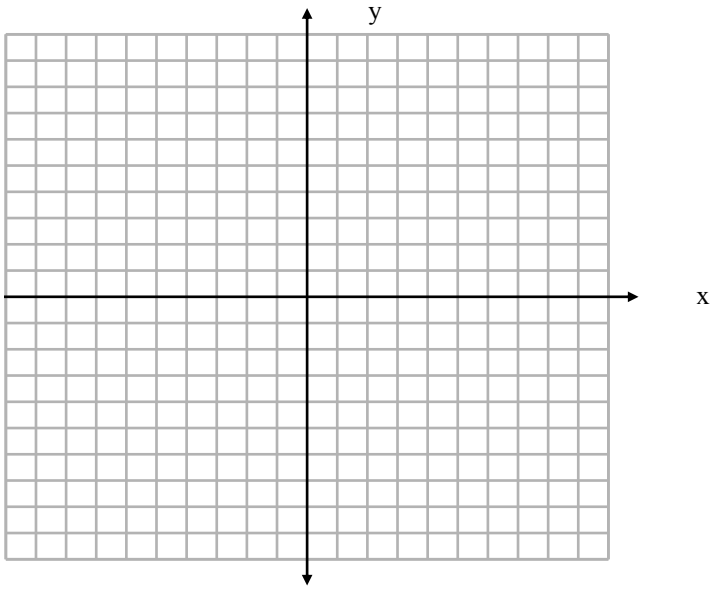


$y = \arctan(x)$

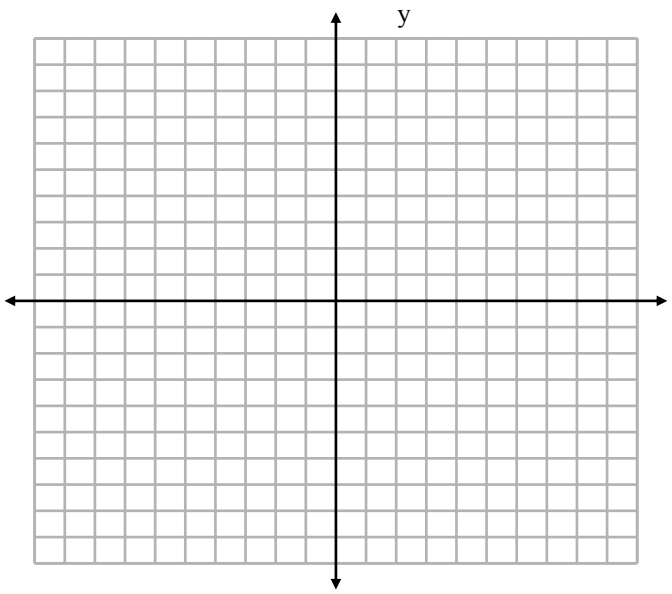


2. Sketch the following graphs on the axes provided.

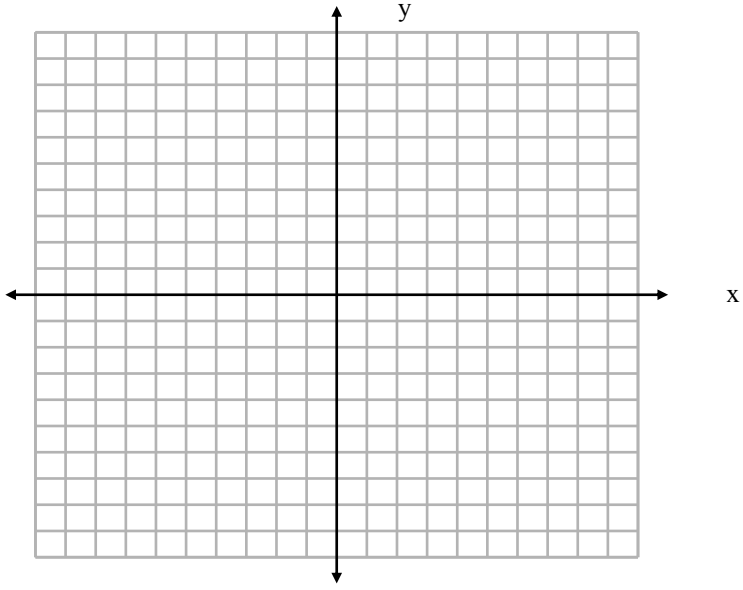
a.) $y = \dfrac{|x|}{x}$



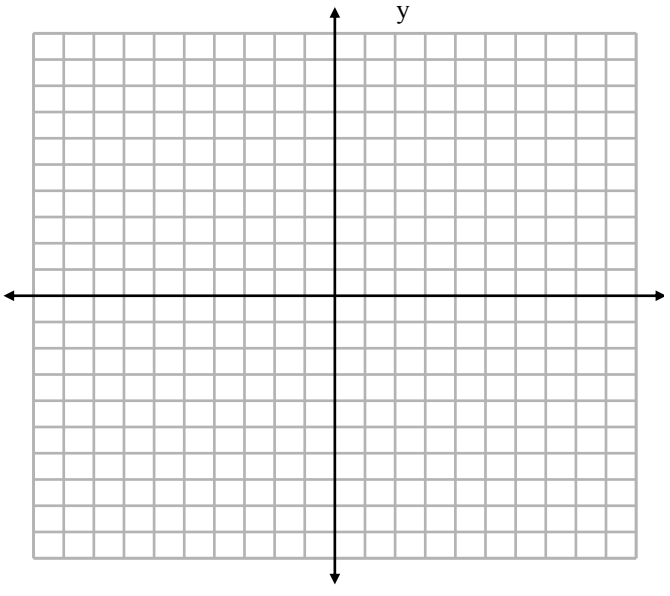
b.) $y = \sqrt{9 - x^2}$



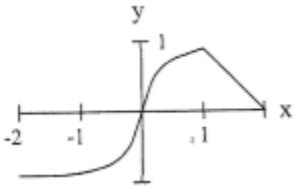
c.) $f(x) = \begin{cases} 2x + 5 & x < -3 \\ x^2 - 4 & -3 \le x < 2 \\ \sqrt{x - 5} & x \ge 5 \end{cases}$



d.) $y = |x^2 - 4|$



3. Given the graph of *f(x)*, sketch the following:



a.) $f(x+1)$ b.) $f(-x)$ c.) $|f(x)|$

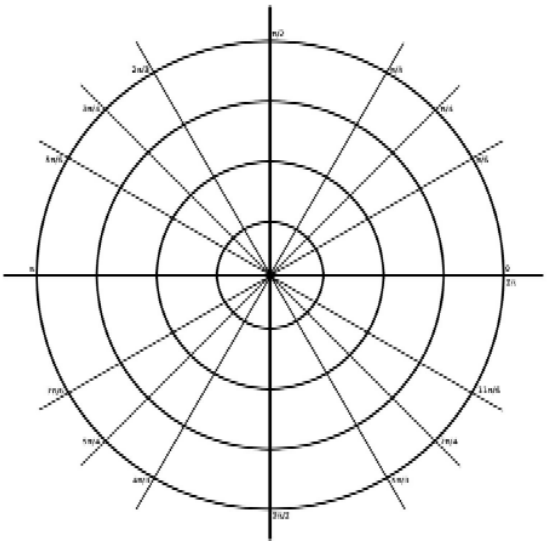
4. Identify any asymptotes (vertical, horizontal, or slant) of each of the following functions:

a.) $f(x)=\dfrac{2}{x-3}$ b.) $f(x)=\dfrac{2(x-1)(x+2)}{x^2-1}$

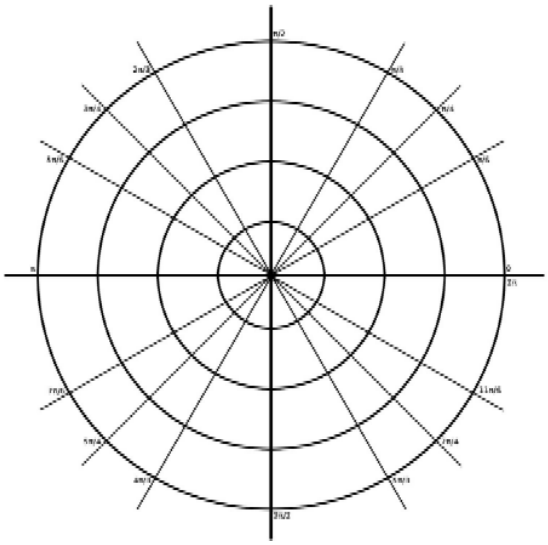
5. Sketch the following polar functions.

a.) $r=5$ b.) $\theta=\dfrac{\pi}{3}$

c.) $r = 2\cos\theta$



d.) $r = 1 - 2\sin\theta$



Part IV: Solving

1. Solve for the indicated variable.

a.) $A = P + nrP$ for *P*

b.) $2x - 2yd = y + xd$ for *d*

2. Solve for *x*.

a.) $4e^{x-2} + 1 = 10$

b.) $\dfrac{\ln(x-1)}{5} = 4$

c.) $\log x - \log(x-6) = 2$

d.) $\dfrac{100}{1+2e^{-.5x}} = 50$

e.) $2\sin\theta + 1 = 0$ on $[0, 2\pi]$

f.) $\cos\theta(1 - 2\sin\theta) = 0$ on $[0, 3\pi]$

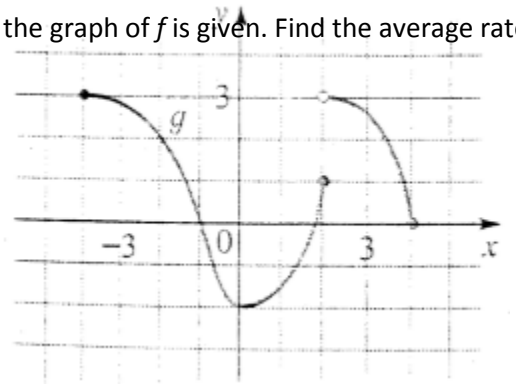
g.) $2\tan\theta = 2\sqrt{3}$

h.) $|3x - 5| < 1$

Part V: Applications

1. Calculate the average rate of change of *f* on the given interval.

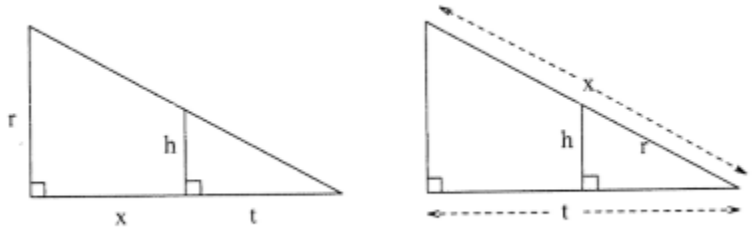
a.) the graph of *f* is given. Find the average rate of change of *f* on the interval $[-4, 2]$



b.) $f(x)=5x^2+2x-3$ on $[2,6]$

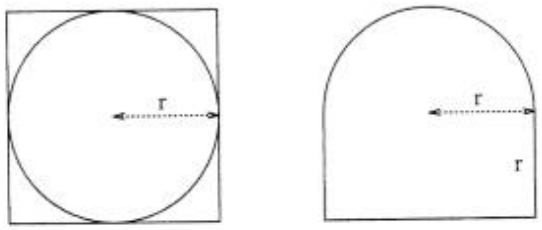
c.) $f(x)=5x^2+2x-3$ on $[a,a+h]$

2. Express x in terms of the other variables in the picture.



(a) (b)

3. (a) Find the ratio of the area inside the square but outside the circle to the area of the square in the picture (a) below.



(a) (b)

(b) Find a formula for the perimeter of a window of the shape in the picture (b) above.

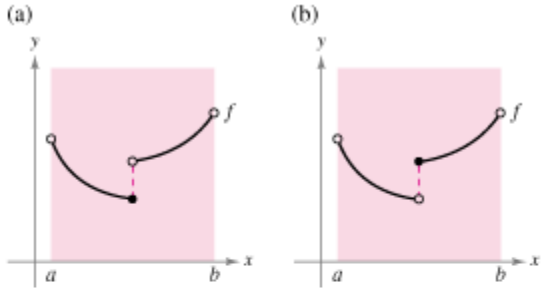
(c) A water tank has the shape of a cone (like an ice cream cone without ice cream). The tank is 10m high and has a radius of 3m at the top. If the water is 5m deep (in the middle) what is the surface area of the top of the water?

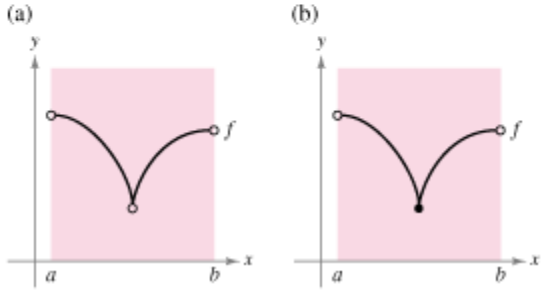
(d) Two cars start moving from the same point. One travels south at 100km/hour, the other west at 50 km/hour. How far apart are they two hours later?

(e) A kite is 100m above the ground. If there are 200m of string out, what is the angle between the string and the horizontal. (Assume that the string is perfectly straight.)

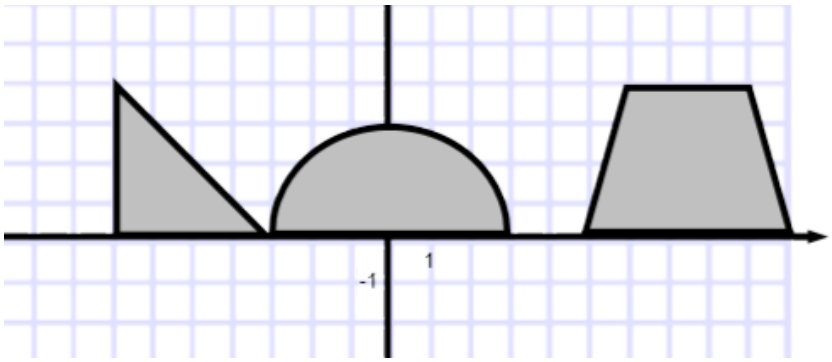
4. Using a graphing calculator, find any relative extrema (min's and max's) of the polynomial:
$f(x) = 3x^4 - 3x^3 - 5x^2 + 6x + 10$.

5. Determine from the graph whether *f* has a minimum on the interval. Write yes or no for each graph.





6. Calculate the area of the shaded region.



7. A curve is traced by a point $P(x, y)$ which moves such that its distance from the point $A(-1,1)$ is three times its distance from the point $B(2,-1)$. Determine the equation of the curve.

8. Find an exponential model of the form $y = ae^{bx}$ that passes through the points $(2,3)$ and $(5,7)$. Use the model to predict the *y*-value when $x = 6$.

9. For each function, find the difference quotient: $\frac{f(x+h)-f(x)}{h}$. Simplify as much as you can.

a.) $f(x) = x^3 + 4x$

b.) $f(x) = 2\sqrt{x+3} - 1$

c.) $f(x) = \frac{2}{x-1}$

d.) $f(x) = \sin x$ (HINT: Use addition identity for sine)

10. Write the partial fraction decomposition of:

a.) $f(x)=\dfrac{1}{x^2-1}$

b.) $f(x)=\dfrac{x^2+12x+12}{x^3-4x}$

11. Complete the square for the following quadratic expressions. Write in the form: $a\left(x-h\right)^2+k$

a.) x^2+6x

b.) $2x^2-18x$

c.) $10x-x^2$

12. Find the n[th] term of the sequence starting at $n=0$

a.) $3,7,11,15,\ldots$

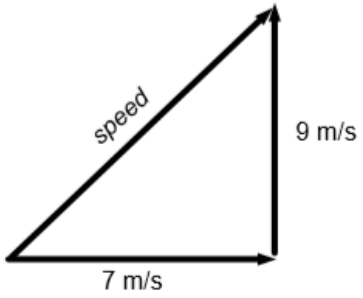
b.) $2,6,18,54,\ldots$

c.) $1,\dfrac{3}{2},\dfrac{5}{6},\dfrac{7}{24},\cdots$

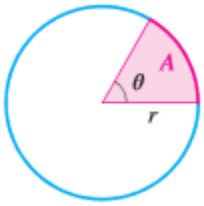
13. Write as a single equation in x and y:

(a) $\begin{cases} x = t+1 \\ y = t^2 - t \end{cases}$ (b) $\begin{cases} x = \sqrt[3]{t} - 1 \\ y = t^2 - t \end{cases}$ (c) $\begin{cases} x = \sin t \\ y = \cos t \end{cases}$

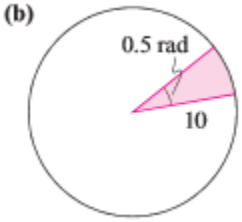
14. A particle is moving along a diagonal line. Its speed in both the horizontal and vertical directions are given. Calculate the speed of the particle in the direction in which it is traveling.



15. Write a formula for the area of a sector of a circle of radius *r* with given angle θ: A = _____



Find the area of the sector shown here:



16. Convert the following coordinates to either rectangular or polar form.

a.) $(5,9) \rightarrow$ polar coordinates

b.) $\left(7, \frac{\pi}{6}\right) \rightarrow$ rectangular coordinates

ANSWER KEY

Part I: Simplifying

1. $\frac{-1}{4(x-4)}$ **2.** $\frac{-1}{x\left(\sqrt{x-1}-\sqrt{x}\right)}$ **3.** $x-6$ **4.** $x-4$ **5.** $\sec x$ **6.** $-\cos\theta$

7. $2\log x+3\log(x-1)-\log(2-x)$ **8.** $\ln\left(\sqrt{\frac{5(x+1)^3}{x^2}}\right)$ **9.** $16x^4$ **10.** $t^{2/3}$

11. $\frac{x^2+2x+6}{(x-1)(x+2)}$ **12.** $\frac{1}{(1+x^2)^{3/2}}$ **13.** 56 **14.** 45 **15.** $n+1$ **16.** $\frac{1}{(2n+3)(2n+2)}$

17. a.) $(2x-9)(2x+9)$ **b.)** $(2x+5)(4x^2-10x+25)$ **c.)** $(x-2)(2x+5)(2x-5)$

d.) $2x^{-1/4}(x-10)$ **18. a.)** $\sum_{n=0}^{17}x^n$ **b.)** $\sum_{n=0}^{20}4x^{2n}$ **19.** $\frac{\sqrt{x^2-9}}{3}$

Part II: Evaluating

1.

Angle (degree)	Angle (radians)	sin(x)	cos(x)	tan(x)
0	0	0	1	0
30	$\frac{\pi}{6}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{3}}$
45	$\frac{\pi}{4}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1
60	$\frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$
90	$\frac{\pi}{2}$	1	0	$\varnothing$

2. a.) 0 **b.)** $-\frac{1}{2}$ **c.)** $\frac{\sqrt{2}}{2}$ **d.)** $\frac{\pi}{3}$ **e.)** $-\frac{\pi}{4}$ **f.)** $\frac{\pi}{3}$ **g.)** $\frac{3}{5}$ **h.)** $\frac{12}{5}$

3. a.) $x^4-6x^3+13x^2-26x+45-\frac{89}{x+2}$ **b.)** $x^2-x+1+\frac{x^2+3}{x^3+1}$ **4. a.)** $210\pi cm^3$ **b.)** $\frac{200}{3}\pi in^3$ **5.**

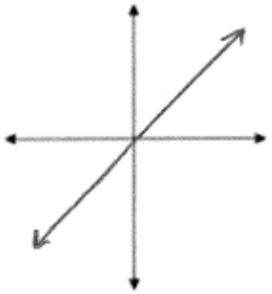
a.) 79 **b.)** $\frac{1}{2}$ **c.)** $\frac{16}{3}$

Part III: Graphing

1.

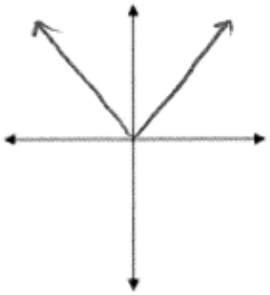
$y = x$

D: $x \in \mathbb{R}$
R: $y \in \mathbb{R}$



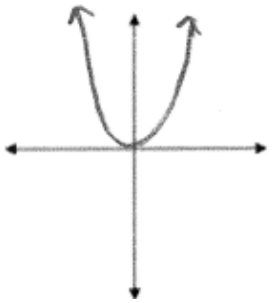
$y = |x|$

D: $x \in \mathbb{R}$
R: $y \in \mathbb{R}$



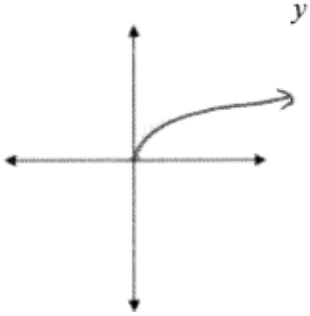
$y = x^2$

D: $x \in \mathbb{R}$
R: $y \geq 0$



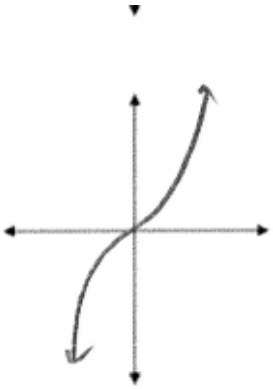
$y = \sqrt{x}$

D: $x \geq 0$
R: $y \geq 0$



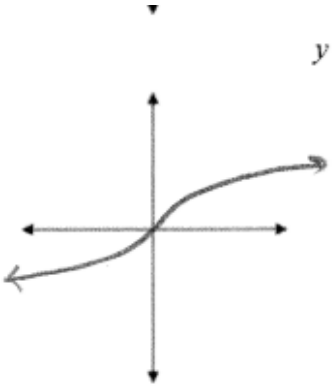
$y = x^3$

D: $x \in \mathbb{R}$
R: $y \in \mathbb{R}$



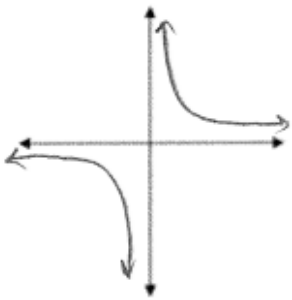
$y = \sqrt[3]{x}$

D: $x \in \mathbb{R}$
R: $y \in \mathbb{R}$



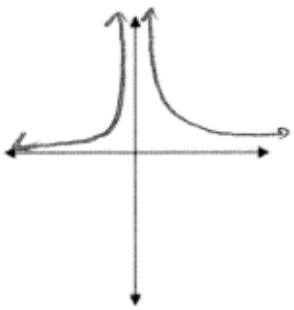
$y = \frac{1}{x}$

D: $x \neq 0$
R: $y \neq 0$
v.a: $x = 0$
h.a: $y = 0$



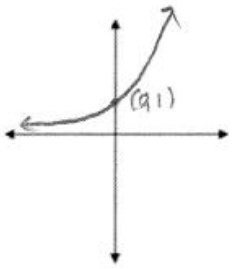
$y = \frac{1}{x^2}$

D: $x \neq 0$
R: $y > 0$
v.a: $x = 0$
h.a: $y = 0$



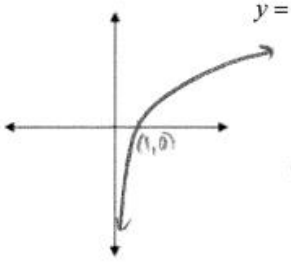
$y = e^x$

D: $x \in \mathbb{R}$
R: $y > 0$
h.a: $y = 0$



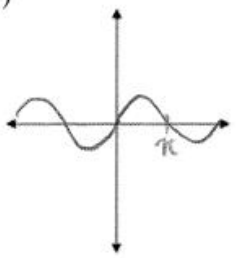
$y = \ln(x)$

D: $x > 0$
R: $y \in \mathbb{R}$
v.a: $x = 0$



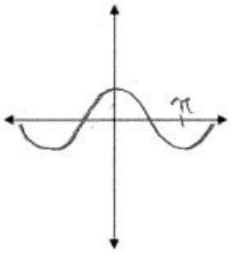
$y = \sin(x)$

D: $x \in \mathbb{R}$
R: $-1 \le y \le 1$



$y = \cos(x)$

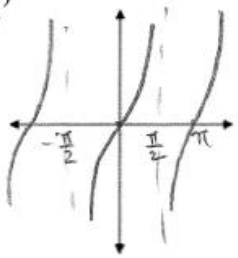
D: $x \in \mathbb{R}$
R: $-1 \le y \le 1$



$y = \tan(x)$

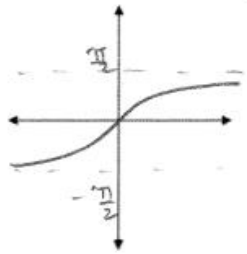
D: $x \neq \frac{(2n+1)\pi}{2}$
R: $y \in \mathbb{R}$
v.a: $x = \frac{(2n+1)\pi}{2}$

$n \in \mathbb{Z}$

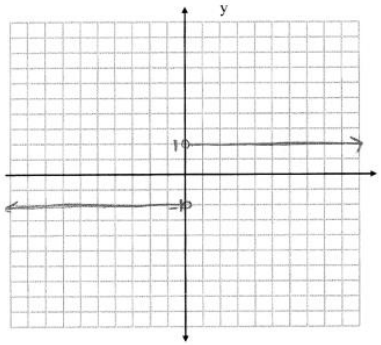


$y = \arctan(x)$

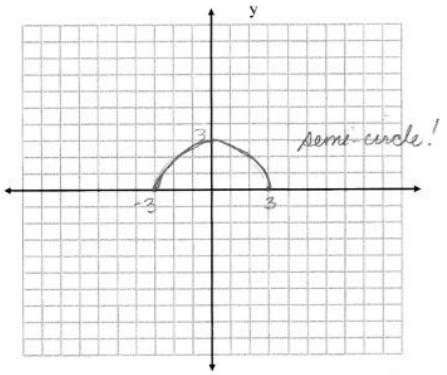
D: $x \in \mathbb{R}$
R: $-\frac{\pi}{2} < y < \frac{\pi}{2}$
h.a: $y = -\frac{\pi}{2}, y = \frac{\pi}{2}$



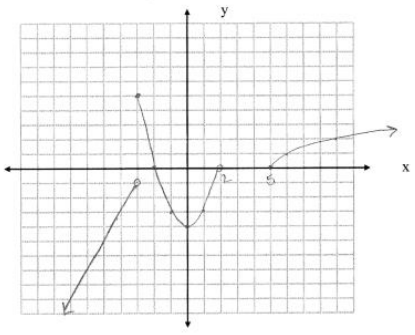
2. a.)



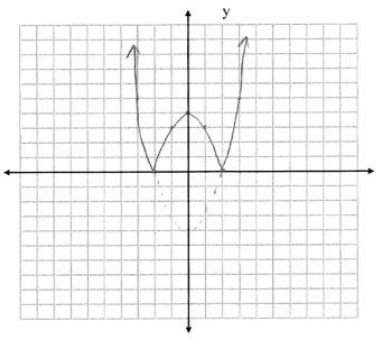
b.)



c.)

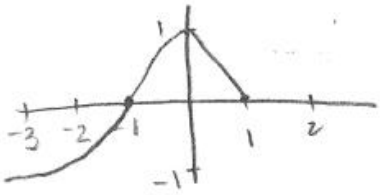


d.)

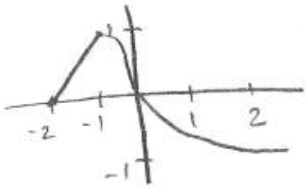


3.

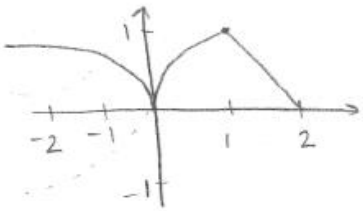
a.) $f(x+1)$ left 1



b.) $f(-x)$ reflect over y-axis



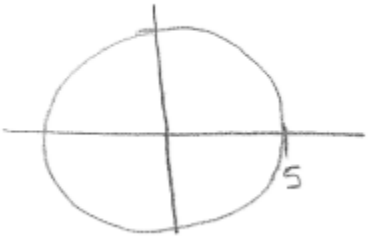
c.) $|f(x)|$ all negative y's become (+)



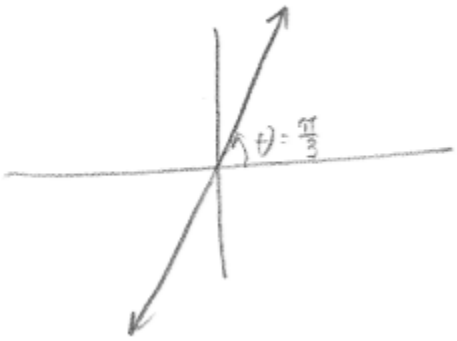
4. a.) $v.a.: x = 3$, $h.a.: y = 0$

b.) $v.a.: x = -1$, $h.a.: y = 2$, $D: x \neq -1, 1$

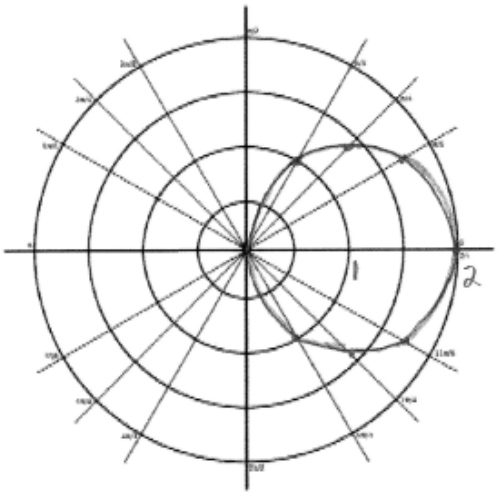
5. a.) $r = 5$



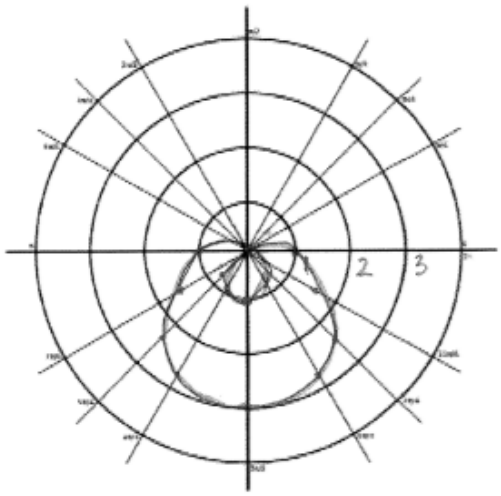
b.) $\theta = \frac{\pi}{3}$



c.) $r = 2\cos\theta$



d.) $r = 1 - 2\sin\theta$



Part IV: Solving

1. a.) $P=\dfrac{A}{1+nr}$ **b.)** $d=\dfrac{2x-y}{x+2y}$ **2. a.)** $x\approx 2.811$ **b.)** $x=e^{20}+1$

c.) $x=\dfrac{600}{99}$ **d.)** $x\approx 1.386$ **e.)** $\theta=\dfrac{7\pi}{6},\dfrac{11\pi}{6}$ **f.)** $\theta=\dfrac{\pi}{6},\dfrac{\pi}{2},\dfrac{5\pi}{6},\dfrac{3\pi}{2},\dfrac{13\pi}{6},\dfrac{5\pi}{2},\dfrac{17\pi}{6}$

g.) $\theta=\dfrac{\pi}{3}+n\pi, n\in Z$ **h.)** $\dfrac{4}{3}<x<2$

Part IV: Applications

1a.) $-\dfrac{1}{3}$ **b.)** 42 **c.)** $10a+5h+2$ **2. a.)** $x=\dfrac{rt}{h}-t$ **b.)** $x=\dfrac{rt}{\sqrt{r^2-h^2}}$

3. a.) $1-\dfrac{\pi}{4}$ **b.)** $P=4r+\pi r$ **c.)** $\dfrac{9\pi}{4}m^2$ **d.)** $100\sqrt{5}km$ **e.)** $\theta=30^\circ$

4. rel min: $f(-0.87)=4.69, f(1.094)=10.949$**, rel. max:** $f(0.526)=11.566$

5. a.) yes, **b.)** no, **a.)** no, **b.)** yes **6.** $24+\dfrac{9\pi}{2}$ **7.** $8x^2+8y^2-38x+20y+43=0$

8. $y=1.705e^{0.282x}$**;** $y(6)\approx 9.284$ **9. a.)** $3x^2+3xh+h^2+4$ **b.)** $\dfrac{2\sqrt{x+h+3}-2\sqrt{x+3}}{h}$

c.) $\dfrac{-2}{(x+h-1)(x-1)}$ **d.)** $\dfrac{\sin x\cos(h)+\sin(h)\cos x-\sin x}{h}$ **10. a.)** $\dfrac{1}{2}\left(\dfrac{1}{x-1}-\dfrac{1}{x+1}\right)$

b.) $\dfrac{-3}{x}+\dfrac{5}{x-2}-\dfrac{1}{x+2}$ **11. a.)** $(x+3)^2-9$ **b.)** $2\left(x-\dfrac{9}{2}\right)^2-\dfrac{81}{2}$ **c.)** $-(x-5)^2+25$

12. a.) $a_n=4n+3$ **b.)** $a_n=2(3)^n$ **c.)** $a_n=\dfrac{2n+1}{(n+1)!}$ **13. a.)** $y=x^2-3x+2$

b.) $y=(x+1)^6-(x+1)^3$ **c)** $x^2+y^2=1$ **14.** $\sqrt{130}\,m/s$ **15.** $A=\dfrac{1}{2}r^2\theta$**,** $A=25$

16. a.) $\left(\sqrt{106},1.064\right)$ **b.)** $\left(\dfrac{7\sqrt{3}}{2},\dfrac{7}{2}\right)$